Continous random Variable

[1] Def: random Variable that take infinity number of Variables

[2] Condition:

* $a < x < b$ or $a \leq x \leq b$ "No difference"
* probability function such that

$\rightarrow f(x) \geq 0$ for all x

$\rightarrow \int_{-\infty}^{\infty} f(x)\, dx = 1 \Rightarrow$ Area under fn $= 1$

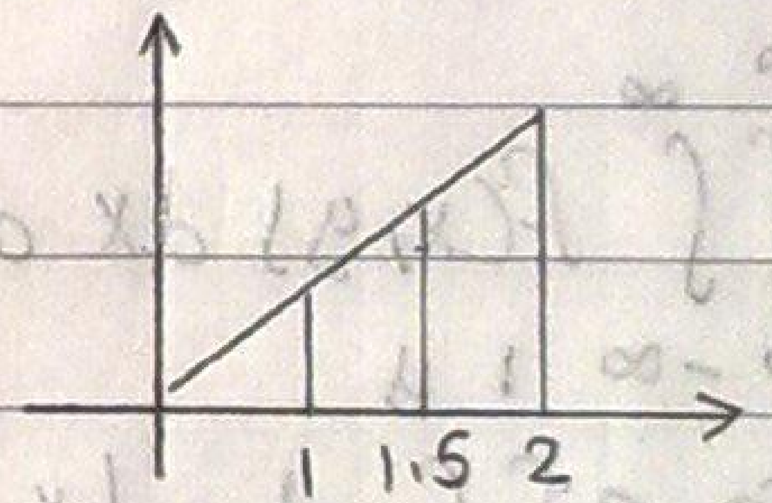
EX: let $f(x)$ be Prob. function $f(x) = \begin{cases} cx & 0 < x < 2 \\ 0 & \text{other wise} \end{cases}$

Find ① c , ② $P(1 \leq x \leq 1.5)$, ③ $E(x)$

① $c \int_0^2 x\, dx = 1 \Rightarrow c\left[\frac{x^2}{2}\right]_0^2 = 1 \Rightarrow c = \frac{1}{2}$

② $P(1 \leq x \leq 1.5) = \frac{1}{2}\int_1^{1.5} x\, dx = \frac{5}{16}$

③ $E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int x^2\, dx = \frac{4}{3}$



[3] Expectation: Average, mean value, expected Value

$$\Rightarrow E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

[4] Marginal probability function:

$f_1(x) = \int_{\text{all } y} f(x,y)\, dy$ & $f_1(y) = \int_{\text{all } x} f(x,y)\, dx$

$\rightarrow$ Used to check independency and solving Problems with double integration

5 Joint Prob. function

① $f(x,y) \geq 0$ ② $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1 \Rightarrow$ Volume $=1$

6 x, y independent

$$f_1(x)\cdot f_2(y) = f(x,y)$$

Ex: P.44

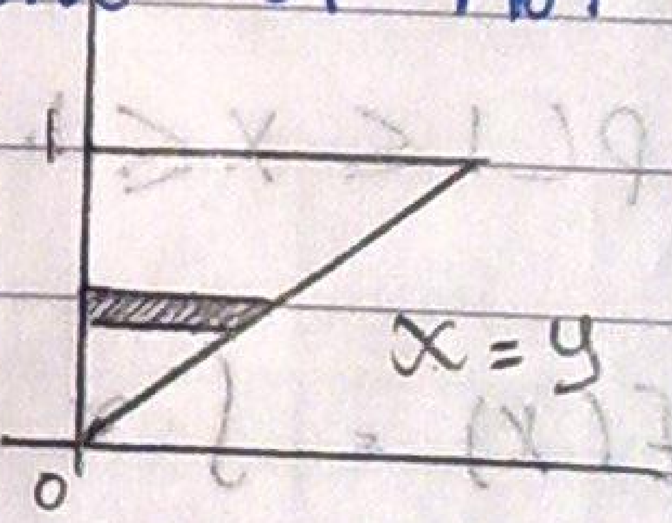
let $f(x,y)$ be joint prob.

$$f(x,y) = \begin{cases} c/y & , 0<x<y \ , \ 0<y<1 \\ 0 & , \text{otherwise} \end{cases}$$

find: ① c ② $P(X>0.5)$ ③ $P(X<\frac{1}{2}, y>\frac{1}{3})$

④ $P(X+y>0.5)$ ⑤ independent or not ??!

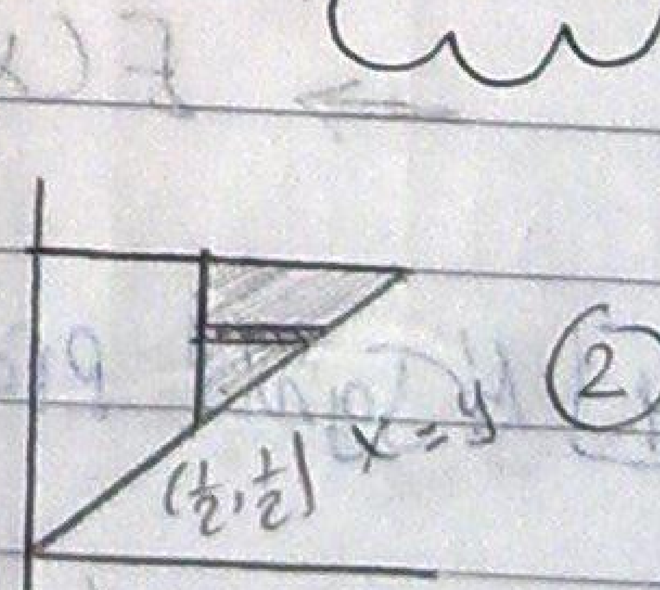
$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$



① $c\int_0^1\int_0^y \frac{1}{y}\,dx\,dy = 1 \Rightarrow c\int_0^1 \frac{x}{y}\Big|_0^y = 1$

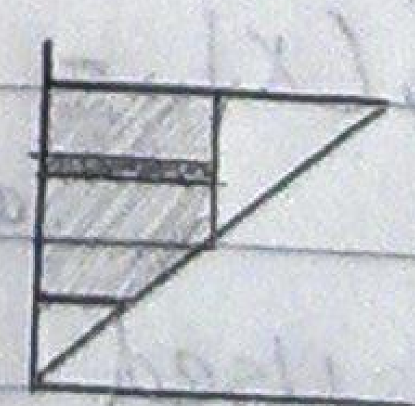
$\Rightarrow c\int_0^1 \frac{1}{y}\cdot y\,dy = 1 \Rightarrow c[y]_0^1 = 1 \Rightarrow \boxed{c=1}$

② $P(X>0.5) = \int_{\frac{1}{2}}^{1}\int_{\frac{1}{2}} \frac{1}{y}\,dx\,dy = 0.53$



③ $P(X>\frac{1}{2}, y>\frac{1}{3}) = \int_{\frac{1}{3}}^{\frac{1}{2}}\int_0^y \frac{1}{y}\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_0^{\frac{1}{2}} \frac{1}{y}\,dx\,dy$

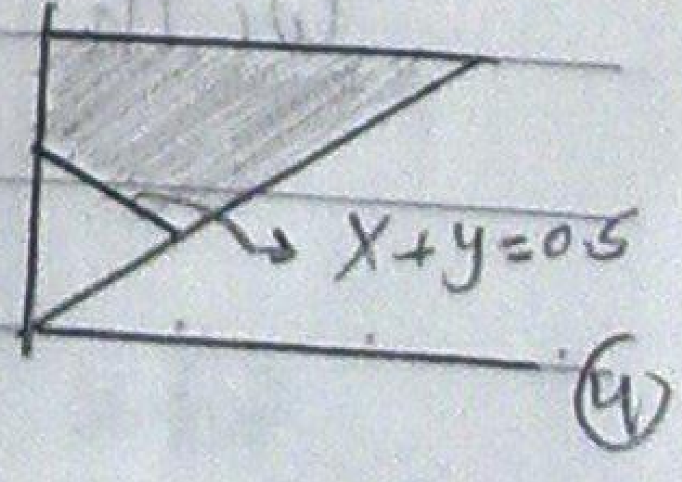
$= 0.513$



④ $P(X+y>0.5) = \int_{\frac{1}{4}}^{\frac{1}{2}}\int_{\frac{1}{2}-y}^{y} \frac{1}{y}\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_0^{y} \frac{1}{y}\,dx\,dy$

⑤ $f_1(x) = \int_x^1 \frac{1}{y}\,dy = -\ln x \quad 0<x<1$

$f_2(x) = \int_0^y \frac{1}{y}\,dx = 1 \quad 0<y<1$



$f(x,y) = \frac{1}{y} \neq f_1(x)\cdot f_2(x) \Rightarrow x \,\&\, y$ not indep.